International Journal of Control, Automation and Systems 15(3) (2017) 1302-1313
http://dx.doi.org/10.1007/s12555-016-0176-6

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

The LQR Baseline with Adaptive Augmentation Rejection of Unmatched Input Disturbance

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Abstract: In this paper, disturbance rejection algorithm based on model reference adaptive control (MRAC) augmentation of linear quadratic regulator (LQR) controller is investigated for uncertain turbulence disturbances. The direct adaptive state feedback optimal controller based on an LDU gain decomposition parameterization is designed to solve the turbulence compensation problem to enhance control performance. Under the proposed control techniques, the bounded stability is achieved and the controller is able to remain within tight bounds on the matched and unmatched uncertainties. Finally, simulation results are presented to illustrate the effectiveness of the proposed MRAC augmentation of LQR controller.

Keywords: Adaptive control, LDU decomposition, LQR basline controller, tracking performance.

NOMENCLATURE

A_p	open-loop state matrix about nominal trim
A	LQR controller based state matrix
B_p	open-loop input matrix about nominal trim
B_d	LQR controller based state matrix uncertainty
B_r	LQR controller based reference input matrix
B	LQR controller based control input matrix
C	open-loop output matrix about nominal trim
$x(t)$	open-loop plant measured state
$\hat{x}(t)$	predictor plant state
$x_{ref}(t)$	reference state
$y(t)$	open-loop plant measured output
$y_m(t)$	the model reference output
$u_{bl}(t)$	the baseline controller
$u_{ad}(t)$	the adaptive controller
$G_p(s)$	LQR controller based transfer matrix
$G_d(s)$	LQR controller based disturbance transfer matrix
Γ	unknown matrix of constant parameters
$\Omega(x)$	the known Lipschitz-continuous regressor vector
$f(x)$	system matched uncertainty
$\xi(t)$	system unmatched uncertainty
Λ	control ineffectiveness uncertainty
$\mathrm{W}_m(s)$	reference transfer function matrix
$\xi_m(s)$	modified interactor matrix
$P_l(s)$	poles polynomial matrix of the system
$Z_0(s)$	zeros polynomial matrix of the system
$Z_d(s)$	zeros polynomial matrix of the disturbance
$d(t)$	input disturbance regressor vector
$d_{j0}, d_{jk}(t)$	some unknown constants of the disturbance
P_{ref}, P_{prd}	symmetric positive-definite matrices of the algebraic Lyapunov equations
$f(t)$	some known bounded continuous disturbance
K_1^{*T}, K_2^* $K_3^*(t)$	the nominal parameters of the adaptive controller
$\Phi^*, \Phi(t)$	the nominal and estimate parameters of the adaptive controller
$\Theta_0^*, \Theta(t)$	the nominal and estimate parameters of matrix L
$\Psi^*, \Psi(t)$	the nominal and estimate parameters of matrix D
$h(s)$	an introduced signal filter
$\xi(t), \eta(t)$ $\zeta(t), m(t)$	introduced auxiliary signal of adaptive laws
$\varepsilon(t)$	introduced estimation error
$\mathrm{P}_\theta, \mathrm{P}_\phi, \mathrm{P}$	the gain of the MRAC adaptive laws
$\mathrm{P}_x, \mathrm{P}_r, \mathrm{P}_\tau$	the gain of the PMRAC adaptive laws
δ_r	rudder deflection angle, deg
δ_a	aileron deflection angle, deg
φ	roll angle, rad
p	roll rate, rad/s
r	yaw rate, rad/s

Manuscript received March 23, 2016; revised July 18, 2016; accepted August 28, 2016. Recommended by Associate Editor Choon Ki Ahn under the direction of Editor Hamid Reza Karimi. This work was supported by Funding of Jiangsu Innovation Program for Graduate Education (grant KYLX0278).

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1. INTRODUCTION

During the past decades, several disturbance attenuation and rejection approaches have been established. The H_∞ control technique which has advantages over classical control techniques is an effective disturbance attenuation method and has already been successfully applied in practice [1, 2]. However, the robustness against disturbance achieved by the H_∞ control approach is guaranteed at the price of degraded nominal performance and the disturbance is assumed to have finite energy. Slide mode control is an effective robust control algorithm since it is insensitive to model uncertainties, external disturbances and parameter variations [3–9]. In [7] a method that combines H_∞ and integral sliding mode control was proposed. The main idea is to choose such a projection matrix, ensuring that unmatched perturbations are not amplified and moreover minimized. However, the slide mode control method has an inherent feature of the chattering phenomenon caused by the high-frequency control switching. This chattering could severely deteriorate the performance of the system. Recently, researches have been done to ensure the robustness for system uncertainties and external disturbances through using adaptive fuzzy output feedback [10–12], dissipativity and $l_2 - l_\infty$ approaches [13–15].

Adaptive control systems, under some generic design conditions, are capable of tolerating large parametric, structural and parameterizable disturbance uncertainties, to ensure desired system asymptotic tracking performance, in addition to system stability [16]. Such asymptotic tracking performance is crucial for many performance-critical system applications such as aircraft control systems. Some adaptive control methods with optimal control design were promoted to solve the disturbance problem [17–24]. In [17], the dead-zone modification stops the adaptation process when the norm of the tracking error becomes smaller than the prescribed value. However, the dead-zone modification is not lipschitz and it may cause high-frequency and other undesirable effects, especially when the tracking error is near the dead-zone boundary. The $\sigma-$modification together with a dynamic normalization was employed in the adaptive law to ensure robustness for small tracking errors and $e-$modification was introduced to replace the constant damping gain σ with a term proportional to a linear combination of the system tracking errors [18, 19]. However, for large tracking errors, the dead zone, the $\sigma-$modification, and the $e-$modification slow down the adaptation. In [20, 21], the adaptive feedforward cancelation algorithms can be applied to reject such frequency-modulated disturbances, which are exactly equivalent to a set of compensators implementing the internal model principle. In [22–24], optimal control modification method was developed for systems with unmatched uncertainty using a predictor model for estimating the control input. However, the existing adaptive disturbance rejection designs are mainly for the matched disturbance rejection or for the unmatched disturbance rejection, but with certain difficulty of achieving the asymptotic output tracking performance.

The LQR baseline controller augmented with adaptive component has proven to be an effective choice for accommodating the parametric uncertainties present in flight control applications and for ensuring satisfactory reference tracking [25–32]. This baseline controller with adaptive augmentation architecture could provide good tracking and transient performance with certain uncertainty. A combining direct and indirect MRAC (CMRAC) was developed to augment the LQR based method and improve the transient performance compared with the direct MRAC method [29]. A comparison was given between the now-classical direct MRAC [34, 35] and CMRAC augment system [30], and the CMRAC augment method has the better transient characteristics than the MRAC method, when using prediction errors in addition to tracking errors, in formulating adaptive law dynamics. In [31], an alternative model reference adaptive control (AMRAC) state-feedback architecture for MIMO systems with matched uncertainties was developed and the tracking performance of both the AMRAC and CMRAC controllers is similar, and both controllers portray smooth control input signals, as compared to MRAC. A predictor-based MRAC (PMRAC) augment method was proposed in [32] and simulation results confirm the fact that the proposed PMRAC controller provides improved transient response compared with MRAC of the closed-loop system, in the presence of matched uncertainties. However, the existing LQR baseline controller augmented with MRAC, CMRAC, AMRAC and PMRAC are mainly for the matched disturbance rejection.

The motivation for studying LQR baseline controller augmented with adaptive component comes from the fact that it is still significant to develop a new LQR based MRAC disturbance rejection techniques to deal with unmatched input disturbances to ensure the asymptotic output tracking performance and improve the transient performance compared with the single MRAC method. The main contributions of this paper are described as follows.

1) A new LQR based MRAC method is introduced to deal with unmatched input disturbances to ensure the asymptotic output tracking and improve the transient performance.

2) The proposed LQR based MRAC with LDU decomposition-based controller is developed for multivariable linear systems with unmatched input disturbances, including key design conditions in terms of system control and disturbance relative degrees, nominal plant-model matching control designs, adaptive law and stability analysis.

3) Comparisons and extensive simulations are studied

to verify the effectiveness of the proposed new LQR based MRAC method.

The rest of this paper is organized as follows. In section 2, a linear time-invariant system problem in matched and unmatched uncertainties is formulated and the LQR baseline controller is designed. In section 3, we propose LQR baseline controller augment with MRAC to solve the disturbance and we illustrate an application of the proposed adaptive design to aircraft wind disturbance rejection control. In section 4 and 5, some simulation results and conclusions are discussed.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. Problem formulation

Consider the linear time-invariant system in the following form:

$$\begin{aligned}&\dot{x}(t)=A_p x(t)+B_p\Lambda(u(t)+f(x))+\xi(t),\\&y(t)=Cx(t), f(x)=\Gamma^T\Omega(x),\end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the system state vector, $A_p\in R^{n\times n}$ and $B_p\in R^{n\times M}$ are constant and known, $C\in R^{M\times n}$ is constant and unknown, $u(t)\in R^M$ is the control input, $\Lambda\in R^{M\times M}$ and $f(x)$ are matched uncertainty, Λ with an unknown constant diagonal positive-definite matrix. $f(x)$ is the linear-in-parameters state-dependent matched uncertainty. $\Gamma\in R^{N\times M}$ is a constant matrix of the unknown coefficients and $\Omega(x)\in R^N$ is the known N-dimensional regressor vector, whose components are locally lipschitz-continuous functions. $\xi(t)\in R^n$ is the unmatched bounded disturbance input, $y(t)\in R^M$ is the regulated output, $x(t)\in R^M$ is the system state.

2.2. LQR control design

When the uncertainties parameter $\Lambda=I_{M\times M}, f(x)=0$ and $\xi(t)=0$, the plant (1) becomes

$$\begin{aligned}&\dot{x}(t)=A_p x(t)+B_p u(t),\\&y(t)=Cx(t).\end{aligned} \tag{2}$$

The LQR controller is

$$u(t)=K_x^T x(t)+K_r^T r(t), \tag{3}$$

where $K_x^T\in R^{M\times n}$ and $K_r^T\in R^{M\times M}$ are the baseline feedback and feedforward gain matrices and $r(t)\in R^M$ is a bounded reference input signal. These gains can be calculated as follows:

$$K_x^T=-R^{-1}B_p^T P, K_r^T=-\left(CA_{ref}^{-1}B_p\right)^{-1}, \tag{4}$$

where the nominal matrix is

$$A_{ref}=A_p+B_pK_x^T, \tag{5}$$

Q is a symmetric positive semidefinite matrix and R is a symmetric positive definite matrix. The cost weight matrices (A_p,B_p) is stabilizable and $\left(A_p,Q^{\frac{1}{2}}\right)$ is detectable. For the infinite-time problem, the optimal steady-state control law for u using state feedback is formed by solving the algebraic Riccati equation using Q and R form (5), the equation is given as

$$PA_p+A_p{}^TP-PB_pR^{-1}B_p^TP+Q=0. \tag{6}$$

The reference model dynamics is

$$\begin{aligned}&\dot{x}(t)=Ax(t)+B_r r(t)\\&A=A_p+B_pK_x^T, B_r=BK_r^T.\end{aligned} \tag{7}$$

If uncertainties and disturbances are not present, the LQR controller is well performed for the system. However, the not adequate performance or stability is provided by the LQR controller when the disturbances are turned on.

2.3. Controller objective

The controller structure is $u(t)=u_{bl}(t)+u_{ad}(t)=K_x^Tx(t)+v(t)$, where $u_{bl}(t)=K_x^Tx(t)$ is used to improve the transient performance of the system and $u_{ad}(t)=v(t)$ is used to ensure the stability of the output tracking in the case of matched and unmatched disturbances. When the parameter $\Lambda\neq I_{M\times M}, f(x)\neq 0, \xi(t)\neq 0$, the reference model dynamics become

$$\begin{aligned}\dot{x}(t)=&\left(A_p+B_p\Lambda K_x^T\right)x(t)+B_p\Lambda(v(t)+f(x))\\&+\xi(t).\end{aligned} \tag{8}$$

We define the system matrix and disturbance as

$$\begin{aligned}&A=A_p+B_p\Lambda K_x^T,\ \ B=B_p\Lambda,\\&B_d d(t)=Bf(x)+\xi(t).\end{aligned} \tag{9}$$

We have the LQR controller based multi-output linear time-invariant system described by

$$\dot{x}(t)=Ax(t)+Bv(t)+B_d d(t)\ \ y(t)=Cx(t), \tag{10}$$

where $A\in R^{n\times n}$, $B\in R^{n\times M}$, $B_d\in R^{n\times p}$ and $C\in R^{M\times n}$, are constant and unknown, and $d(t)=[d_1(t),\ldots,d_p(t)]^T\in R^p$ is the disturbance vector. The element $d_j(t)$ is characterized as

$$d_j(t)=d_{j0}(t)+\sum_{k=1}^{q_j}d_{jk}f_{jk}(t), \tag{11}$$

where d_{j0} and d_{jk} are some unknown constants, f_{jk} are some known bounded continuous signal, $j=1,2,\ldots,p, k=1,2,\ldots q_j$. Note that such a parameterizable disturbance feature is necessary for an adaptive compensation design to cancel the disturbance effect.

An augmentation control signal $v(t)$ is introduced to cope with system parameter uncertainties. The state vector x(t) is available for measurement, the nominal state feedback controller with the disturbance rejection term is

$$v(t)=v^*(t)=K_1^{*T}x(t)+K_2^*r(t)+K_3^*(t), \tag{12}$$

where the nominal parameters $K_1^{*T}\in R^{M\times n}$ and $K_2^{*T}\in R^{M\times M}$ are for the plant-model output matching and $K_3^*(t)\in R^M$ is used to cancel the effect of the disturbance

$d(t)$. The control objective is to design an adaptive controller $v(t)$ so that the baseline controller based system (10) output state vector signal $y(t)$ can asymptotically track a reference output vector signal $y_m(t)$ generate from a chosen reference model:

$$y_m(t) = W_m(s)[r](t), \tag{13}$$

where $W_m(s) \in R^{M\times M}$ is a stable transfer function matrix to be chosen as $W_m(s) = \xi_m^{-1}(s)$ for the modified interactor matrix $\xi_m(s)$ of $G_p(s) = C(sI-A)^{-1}B$. Note that in this paper, we use the notation $y = G(s)[u](t)$ to represent the output $y(t)$ of a system whose transfer matrix is $G(s)$ and input is $u(t)$, a convenient notation for adaptive control systems.

2.4. Preliminaries and assumptions

Lemma 1 [33]**:** For every real matrix with nonzero leading principal minors can be uniquely factored as

$$K_p = LDU, \tag{14}$$

where L is unity lower triangular, U is unity upper triangular, and

$$\begin{aligned} D &= diag\{d_1, d_2, \ldots, d_m\} \\ &= diag\{\Delta_1, \Delta_2\Delta^{-1}, \ldots, \Delta_m\Delta_{m-1}^{-1}\}. \end{aligned} \tag{15}$$

Lemma 2 [13]**:** For any $M\times M$ strictly proper and full rank rational transfer matrix $G(s)$, there exists a lower triangular polynomial matrix $\xi_m(s)$, defined as the left interactor matrix of $G(s)$, of the form

$$\xi_m(s) = \begin{bmatrix} d_1(s) & 0 & 0 & 0 \\ h_{21}^m(s) & d_2(s) & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ h_{M1}^m(s) & h_{M2}^m(s) & \ldots & d_M(s) \end{bmatrix}, \tag{16}$$

where $h_{ij}^m(s)$, $j = 1, 2, ..., M-1$, $i = 2, ..., M$, are some polynomials and $d_i(s)$ are any chosen monic stable polynomials such that the high-frequency gain matrix of $G(s)$ defined as $K_p = \lim_{s\to\infty}\xi_m(s)G(s)$.

From the baseline controller based system (10), the control and disturbance transfer functions are obtained as $G_p(s) = C(sI-A)^{-1}B$ and $G_d(s) = C(sI-A)^{-1}B_d$ and are expressed in their left coprime polynomial matrix decompositions: $G_p(s) = P_l^{-1}(s)Z_p(s)$ and $G_d(s) = P_l^{-1}(s)Z_d(s)$, where $P_l(s), Z_p(s) \in R^{M\times M}$ and $Z_d(s) \in R^{M\times p}$ are some polynomial matrices.

Assumption 1: All zero of G_p are stable, and (A,B,C) is stabilizable and detectable.

Assumption 2: $G_p(s)$ is strictly proper with full rank and has a known modified interactor matrix $\xi_m(s)$ such that $K_p = \lim_{s\to\infty}\xi_m(s)G_p(s)$ is finite and nonsingular (so that $W_m(s) = \xi_m^{-1}(s)$ can be chosen as the transfer matrix for the reference model system).

Assumption 3: The leading principal minors of the high-frequency gain matrix K_p are nonzero, and their signs are known.

Assumption 4: The transfer matrix $Z_p^{-1}(s)Z_d(s)$ is proper.

Remark 1: Assumption 1 is for output matching and internal signal stability. Assumption 2 is for choosing the reference system model for adaptive control. Assumption 3 is for designing adaptive parameter update laws. Assumption 4 is the relative degree condition from the control input $v(t)$ and the disturbance input $d(t)$ to the output $y(t)$ for the design of a derivative-free disturbance rejection scheme.

3. LQR BASLINE WITH ADAPTIVE AUGMENTATION DISTURBANCE REJECTION CONTROOLER DESIGN

3.1. Adaptive Augmentation Design

In this section, an adaptive rejection of unmatched input disturbances in multivariable systems is introduced to the augmentation of the baseline controller based on LDU decompositions of K_p.

Lemma 3: The matrix $K_d = \lim_{s\to\infty}\xi_m(s)G_d(s)$ is finite if $Z_p^{-1}(s)Z_d(s)$ is proper.

Proof: From Assumption 2, $\lim_{s\to\infty}\xi_m(s)G_p(s) = K_p$ is finite and nonsingular. we have

$$\lim_{s\to\infty} K_p^{-1}\xi_m(s)G_p(s) = I. \tag{17}$$

Hence, if $Z_p^{-1}(s)Z_d(s)$ is proper, $K_p^{-1}\xi_m(s)G_p(s)Z_p^{-1}(s)$ $Z_d(s)$ is proper, that is

$$\lim_{s\to\infty} K_p^{-1}\xi_m(s)G_p(s)Z_p^{-1}(s)Z_d(s) < \infty. \tag{18}$$

Using $G_p(s) = P_l^{-1}(s)Z_p(s)$ in (18), we have

$$\begin{aligned} &\lim_{s\to\infty} K_p^{-1}\xi_m(s)P_l^{-1}(s)Z_p(s)Z_0^{-1}(s)Z_d(s) \\ &= \lim_{s\to\infty} K_p^{-1}\xi_m(s)P_l^{-1}(s)Z_d(s) \\ &= \lim_{s\to\infty} K_p^{-1}\xi_m(s)G_d(s) < \infty. \end{aligned} \tag{19}$$

So that we obtain the following: $\xi_m(s)C(sI-A)^{-1}B_d$ is proper, that is $K_d = \lim_{s\to\infty}\xi_m(s)G_d(s)$ is finite.

Based on Lemma 3, the existence of a nominal controller for the system (10) is established a follows.

Theorem 1: From the baseline controller based system (10) in the unmatched disturbances, under Assumptions 1 and 4, there exists a state feedback control law, to make the roundedness of all closed-loop signals, disturbance rejection, and output tracking the reference $y_m(t)$.

Proof: From the baseline controller based system (10), the input-output form is obtained as

$$y(t) = G_p(s)[v](t) + \bar{y}(t), \tag{20}$$

with $G_p(s) = C(sI-A)^{-1}B$ and $\bar{y}(t) = G_d(s)[d](t) = C(sI-A)^{-1}B_d[d](t)$. Operate the interactor matrix (a polynomial matrix) $\xi_m(s)$ on in the system (10), $\dot{x}(t) =$

$A(x)+Bv(t)+B_d d(t), y(t)=Cx(t)$, to reach an expression of $\xi_m(s)[y_m](t)$ in a possible form:

$$\begin{aligned}\xi_m(s)[y](t) = &-\bar{K}_0 x(t)+\bar{K}_p v(t)+\bar{K}_{p1}\dot{v}+\cdots\\ &+\bar{K}_{pl0}v^{(l_0)}(t)+\bar{K}_d d(t)+\bar{K}_{d1}\dot{d}(t)\\ &+\cdots+\bar{K}_{dl1}d^{(l1)}(t),\end{aligned} \quad (21)$$

with some constant matrices $\bar{K}_0 \in R^{M\times n}, \bar{K}_p \in R^{M\times M}, \bar{K}_{pj} \in R^{M\times M}, j=1,2,\ldots,l_0, \bar{K}_d \in R^{M\times p}$ and $\bar{K}_{di} \in R^{M\times p}, i=1,2,\ldots,l_1$, for some integers $l_0, l_1 \geq 0$. From (10) and (20), we have

$$x(s) = (sI-A)^{-1}Bv(s)+(sI-A)^{-1}B_d d(s), \quad (22)$$

Expressing (21) in s domain and using (22), we have

$$\begin{aligned}\xi_m(s)y(s) = &-\bar{K}_0(sI-A)^{-1}Bv(s)+\bar{K}_p v(s)\\ &+\bar{K}_{p1}sv(s)+\cdots+\bar{K}_{pl_0}s^{l_0}v(s)\\ &-\bar{K}_0(sI-A)^{-1}B_d d(s)+\bar{K}_d d(s)\\ &+\bar{K}_{d1}sd(s)+\cdots+\bar{K}_{dl_1}s^{l_1}d(s).\end{aligned} \quad (23)$$

From Assumption 2 that $K_p = \lim_{s\to\infty}\xi(s)G_p(s)$ is finite and nonsingular and Assumption 4, $K_{pj}=0, j=1,\cdots,l_0, \bar{K}_P = K_p$,and $K_{dj}=0, j=1,\cdots,l_0, \bar{K}_d = K_d$ Hence, we have

$$\xi_m(s)[y](t) = -\bar{K}_0 x(t)+K_p v(t)+K_d d(t). \quad (24)$$

From (24) that the control law can be designed as

$$v(t) = v^*(t) = K_1^{*T}x(t)+K_2^* r(t)+K_3^*(t), \quad (25)$$

where $K_1^{*T} = K_p^{-1}\bar{K}_0, K_2^* = K_p^{-1}$, and $K_3^*(t) = K_{3d}d(t)$ with $K_{3d}(t) = -K_p^{-1}K_d$, which leads the output matching: $\xi_m(s)[y](t) = r(t)$. From (25) to the system (10), we have

$$\begin{aligned}y(t) &= C(sI-A-BK_1^{*T})^{-1}BK_2^*[r](t)\\ &+C(sI-A-BK_1^{*T})^{-1}B[K_3^*](t)\\ &+C(sI-A-BK_1^{*T})^{-1}B_d d(s)\\ &= W_m(s)[r](t) = y_m(t).\end{aligned} \quad (26)$$

Remark 2: From (26), we can conclude that the plant-model matching conditions are:

$$\begin{aligned}&C(sI-A-BK_1^{*T})^{-1}BK_2^* = W_m(s)\\ &W_m(s)K_2^{*-1}K_3^*(s)+C(sI-A-BK_1^{*T})^{-1}B_d d(s)\\ &\quad =0.\end{aligned} \quad (27)$$

3.2. Parameterizations of the Term $K_3^*(t)$

For the disturbance vector $d(t) \in R^p$, each element $d_j(t)$ in (10) can be expressed as

$$d_j(t) = d_{j0}+\sum_{k=1}^{q_j} d_{jk}f_{jk}(t) = \mu_j^{*T}f_j(t), \quad j=1,2,\cdots,p, \quad (28)$$

where the parameter matrix and the disturbance signal components are

$$\mu_j^* = [d_{j0}, d_{j1}, \cdots, d_{jq_j}]^T \in R^{q_j+1}, \quad (29)$$

$$f_j(t) = [1, f_{j1}(t), \cdots, f_{jq_j}(t)]^T \in R^{q_j+1} \quad j=1,2,\ldots,p. \quad (30)$$

Hence, the disturbance $d(t)$ is expressed as

$$d(t) = \mathrm{N}^{*T}f(t), \quad (31)$$

$$\mathrm{N}^{*T} = \begin{bmatrix} \mu_1^{*T} & 0^T_{(q_2+1)} & \cdots & 0^T_{(q_p+1)}\\ 0^T_{(q_1+1)} & \mu_2^{*T} & \cdots & 0^T_{(q_p+1)}\\ \vdots & \vdots & \vdots & \vdots\\ 0^T_{(q_1+1)} & 0^T_{(q_2+1)} & \cdots & \mu_p^{*T}\end{bmatrix} \in R^{p\times q}, \quad (32)$$

$$\begin{aligned}&f(t) = \begin{bmatrix} f_1^T(t) & f_2^T(t) & \ldots & f_p^T(t)\end{bmatrix}^T \in R^q,\\ &q = q_1+q_2+\cdots+q_p+p.\end{aligned} \quad (33)$$

With $K_{3d}^* = \left[k_{3d1}^*, k_{3d2}^*, \ldots, k_{3dp}^*\right], k_{3dj}^* \in R^M$, $j=1, 2, \ldots, p$, the disturbance rejection term $K_3^*(t)$ is parameterized as

$$K_3^*(t) = K_{3d}^*d(t) = K_{3d}^*\mathrm{N}^{*T}f(t) = K_{3f}^*f(t), \quad (34)$$

where the parameter matrix is

$$\begin{aligned}&K_{3f}^* = [\phi_{31}^*, \phi_{32}^*, \ldots, \phi_{3p}^*] \in R^{M\times q},\\ &q = q_1+q_2+\cdots+q_p+p,\\ &\phi_{3j}^* = k_{3dj}^*\mu_j^{*T} \in R^{M\times(q_j+1)}, j=1,2,\ldots,p.\end{aligned} \quad (35)$$

Next, the adaptive disturbance rejection design for the state feedback control scheme will be studied for the plant with uncertainties from the plant and unmatched disturbances.

3.3. Error equation

Applying (12) to the system (10), the closed-loop system becomes

$$\begin{aligned}\dot{x}(t) &= \left(A+BK_1^{*T}\right)x(t)+BK_2^*r(t)\\ &+BK_3^*(t)+B_d d(t)+B\left[v(t)-K_1^{*T}x(t)\right.\\ &\left.-K_2^*r(t)-K_3^*(t)\right],\\ y(t) &= Cx(t).\end{aligned} \quad (36)$$

In view of (13) and (27) the output tracking error equation is

$$\begin{aligned}e(t) &= y(t)-y_m(t)\\ &= W_m(s)K_p^*\left[v-K_1^{*T}x-K_2^*r-K_3^*\right](t)\\ &+f_p(t),\ K_p^* = K_2^{*-1},\end{aligned} \quad (37)$$

where $f_p(t) = Ce^{(A+BK_1^{*T})t}x(0)$ converges to zero exponentially fast due to the stability of $A+BK_1^{*T}$ and $W_m(s) = \xi_m^{-1}(s)$. Hence we have

$$\begin{aligned}\xi_m(s)[e](t) = &K_p\left(v(t)-K_1^{*T}x(t)-K_2^*r(t)\right.\\ &\left.-K_3^*(t)\right).\end{aligned} \quad (38)$$

3.4. Adaptive designs using LDU decomposition

To deal with the uncertainty of the high-frequency gain matrix K_p, we use the LDU decomposition in Lemma 1, we have

$$\begin{aligned}L^{-1}\xi_m(s)[e](t) = &DU\left[v(t)-K_1^{*T}x(t)\right.\\ &\left.-K_2^*r(t)-K_3^*(t)\right].\end{aligned} \quad (39)$$

We have the following formation:

$$Uv(t) = v(t) - (I-U)\,v(t). \tag{40}$$

With (39) and (40), we have

$$\begin{aligned} &L^{-1}\xi_m(s)[e](t) \\ &= D\left[v(t) - (I-U)\,v(t)\right. \\ &\quad \left. -U\left(K_1^{*T}x(t) - K_2^* r(t) - K_3^*(t)\right)\right]. \end{aligned} \tag{41}$$

We have a new parameterization:

$$L^{-1}\xi_m(s)[e](t) = D\left[v(t) - \Phi_0^* v(t) - \Phi_1^{*T}\omega(t)\right], \tag{42}$$

where $\Phi_1^{*T} = [UK_1^{*T},\ UK_2^*,\ UK_{3f}^*]$ and $\omega(t) = [x^T(t),\ r^T(t),\ f^T(t)]^T$. This new parameterization motivates the new controller structure:

$$v(t) = \Phi_0 v(t) + \Phi_1^T\omega(t), \tag{43}$$

where Φ_0 and Φ_1^T are the estimates of Φ_0^* and Φ_1^{*T}, Φ_0 is upper triangular with zero diagonal elements (only its nonzero elements are estimated). Matrix Φ_0 has the same strictly form as that of $\Phi_0^* = (\mathrm{I}-\mathrm{U})$, and

$$\Phi_0 = \begin{bmatrix} 0 & \phi_{12} & \phi_{13} & \dots & \phi_{1M} \\ 0 & 0 & \phi_{23} & \dots & \phi_{2M} \\ \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & \cdots & 0 & \phi_{M-1M} \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in R^{M\times M}. \tag{44}$$

From (42) and (43), we obtain a new error model

$$\xi_m(s)[e](t) + \Theta_0^*\xi_m(s)[e](t) = D\tilde{\Phi}^T(t)\bar{\omega}(t), \tag{45}$$

where the parameter error is $\tilde{\Phi}(t) = \Phi(t) - \Phi^*$, and $\Phi^T(t) = [\Phi_0(t), \Phi_1^T(t)]$ is the estimate of unknown parameter matrix $\Phi^{*T} = [\Phi_0^*, \Phi_1^{*T}], \bar{\omega}(t) = [v^T(t), \omega^T(t)]^T$ and $\omega(t) = \left[x^T(t), r^T(t), f^T(t)\right]^T$. Where $\Theta_0^* = \left(L^{-1} - I\right)$ is introduced to parameterize the unknown matrix L, which has the special form:

$$\Theta_0^* = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \theta_{21}^* & 0 & \dots & 0 \\ \vdots & \dots & 0 & 0 \\ \theta_{M1}^* & \dots & \theta_{MM-1}^* & 0 \end{bmatrix} \in R^{M\times M}. \tag{46}$$

For such a matrix Θ_0^* the parameter vectors defined as

$$\begin{aligned} &\theta_2^* = \theta_{21}^* \in R, \\ &\theta_3^* = [\theta_{31}^*, \theta_{32}^*]^T \in R^2, \\ &\vdots \\ &\theta_{M-1}^* = \left[\theta_{M-11}^*, \dots, \theta_{M-1M-2}^*\right] \in R^{M-2}, \\ &\theta_M^* = \left[\theta_{M1}^*, \dots, \theta_{MM-1}^*\right] \in R^{M-1}, \end{aligned} \tag{47}$$

and their estimates are

$$\begin{aligned} &\theta_2(t) = \theta_{21}(t) \in R, \\ &\theta_3(t) = [\theta_{31}(t), \theta_{32}(t)]^T \in R^2, \\ &\vdots \\ &\theta_{M-1}(t) = \left[\theta_{M-11}(t), \dots, \theta_{M-1M-2}(t)\right] \in R^{M-2}, \\ &\theta_M(t) = \left[\theta_{M1}(t), \dots, \theta_{MM-2}(t)\right] \in R^{M-1}. \end{aligned} \tag{48}$$

We introduce a filter $h(s) = \frac{1}{f(s)}$, where $f(s)$ is chosen as a stable and monic polynomial whose degree is equal to the maximum degree of the modified interactor $\xi_m(s)$. Operating both sides of (45) by $h(s)I_M$ leads to

$$\begin{aligned} &\xi_m(s)h(s)[e](t) + \Theta_0^*\xi_m(s)h(s)[e](t) \\ &= D^*h(s)\left[\tilde{\Phi}^T\bar{\omega}\right](t). \end{aligned} \tag{49}$$

We defined

$$\bar{e}(t) = \xi_m(s)h(s)[e](t) = [\bar{e}_1(t), \dots, \bar{e}_M(t)]^T, \tag{50}$$

$$\eta_i(t) = [\bar{e}_1(t), \cdots, \bar{e}_{i-1}(t)]^T \in R^{i-1}, \quad i = 2,3,\dots,M. \tag{51}$$

From (49) and (50) in (51), we obtained:

$$\begin{aligned} &\bar{e}(t) + \left[0, \theta_2^{*T}(t)\,\eta_2(t), \theta_3^{*T}(t)\,\eta_3(t),\right. \\ &\quad \left.\dots, \theta_M^{*T}(t)\,\eta_M(t)\right]^T \\ &= Dh(s)[\tilde{\Phi}^T\bar{\omega}](t). \end{aligned} \tag{52}$$

Based on the parameterized error equation (49), an estimation error signal is introduced:

$$\begin{aligned} \varepsilon(t) =& \bar{e}(t) + [0, \theta_2^T(t)\,\eta_2(t), \theta_3^T(t)\,\eta_3(t), \\ &\dots, \theta_M^T(t)\,\eta_M(t)]^T + \Psi(t)\xi(t) \in R^M, \end{aligned} \tag{53}$$

where $\Psi(t) \in R^{M\times M}$ is the estimate of $\Psi^* = D$ and

$$\zeta(t) = h(s)[\bar{\omega}](t), i = 1,2,\cdots,M, \tag{54}$$

$$\xi(t) = \Phi^T(t)\zeta(t) - h(s)\left[\Phi^T\bar{\omega}\right](t), \quad i = 1,2,\cdots,M. \tag{55}$$

It then follows from (49), (53), (54) and (55)

$$\begin{aligned} \varepsilon(t) =& [0, \tilde{\theta}_2^T\eta_2(t), \tilde{\theta}_3^T\eta_3(t), \dots, \tilde{\theta}_M^T\eta_M(t)]^T \\ &+ D\tilde{\Phi}^T + \tilde{\Psi}(t)\xi(t), \end{aligned} \tag{56}$$

where $\tilde{\Psi}(t) = \Psi(t) - \Psi^*$ and $\tilde{\theta}_i(t) = \theta_i(t) - \theta_i^*$ are the parameter errors. This error model is choice for update laws. Adaptive laws. Based on the error model (56), the adaptive laws are chosen as

$$\dot{\theta}_i(t) = -\frac{\mathrm{P}_{\theta_i}\varepsilon_i(t)\eta_i(t)}{m^2(t)}, i = 2,\dots,M, \tag{57}$$

$$\dot{\Phi}^T(t) = -\frac{\mathrm{P}_\phi\varepsilon(t)\zeta^T(t)}{m^2(t)}, \tag{58}$$

$$\dot{\Psi}(t) = -\frac{\mathrm{P}\varepsilon(t)\xi^T(t)}{m^2(t)}, \tag{59}$$

where $\mathrm{P}_{\theta_i} = \mathrm{P}_{\theta_i}{}^T > 0$, $i = 2, 3, \dots, M$, and $\mathrm{P} = \mathrm{P}^T > 0$ are adaptive gains; The sign of P_ϕ is defined in (15). $\varepsilon(t) = [\varepsilon_1(t), \varepsilon_2(t), \dots, \varepsilon_M(t)]^T$ is calculated from (56), and

$$m^2(t) = 1 + \zeta^T(t)\zeta(t) + \xi^T(t)\xi(t) + \sum_{i=2}^{M}\eta_i^T\eta_i(t). \tag{60}$$

3.5. Stability analysis

For the adaptive laws (57)-(59), we have following desired stability properties.

Lemma 4 [34]**:** the adaptive laws ensures that

(i) $\theta_i(t) \in L^\infty, i=2,3,\dots,M, \Phi(t) \in L^\infty, \Psi(t) \in L^\infty$, and $\frac{\varepsilon(t)}{m(t)} \in L^2 \cap L^\infty$;
(ii) $\dot{\theta}_i(t) \in L^2 \cap L^\infty, i=2,3,\dots,M, \dot{\Phi}(t) \in L^2 \cap L^\infty$, and $\dot{\Psi}(t) \in L^2 \cap L^\infty$.

Proof: We choose the positive definition function

$$V = \frac{1}{2}\left(\sum_{i=2}^{M} \tilde{\theta}_i^T \Gamma_{\theta_i}^{-1} \tilde{\theta} + tr\left[\tilde{\Psi}^T \Gamma^{-1} \tilde{\Psi}\right] \ + tr\left[\tilde{\Phi}^T D \tilde{\Phi}\right]\right). \tag{61}$$

From(57)-(59), we derive the time-derivate of V

$$\begin{aligned} V =& -\sum_{i=2}^{M} \frac{\tilde{\theta}_i^T \varepsilon_i(t)\eta_i(t)}{m^2(t)} - \frac{\xi^T(t)\tilde{\Psi}\varepsilon(t)}{m^2(t)} \\ & - \frac{\zeta^T(t)\tilde{\Phi}D\varepsilon(t)}{m^2(t)} \\ =& -\frac{\varepsilon^T(t)\varepsilon(t)}{m^2(t)} \leq 0. \end{aligned} \tag{62}$$

Similar to the case in [34], we derive that $\theta_i(t) \in L^\infty, i=2,3,\dots,M, \Phi \in L^\infty, \Psi \in L^\infty, \frac{\varepsilon(t)}{m(t)} \in L^2 \cap L^\infty$, $\theta_i(t) \in L^2 \cap L^\infty, i=2,3,\dots,M, \dot{\Phi}(t) \in L^2 \cap L^\infty$, and $\dot{\Psi}(t) \in L^2 \cap L^\infty$.

Based on Lemma 4, the following desired closed-loop system properties are established.

Theorem 2: For the plant (10) with uncertainties from the system parameters and disturbance (1) under Assumptions 1-4, and the reference model (13), the LDU decomposition based MRAC scheme with the adaptive controller (12) and adaptive parameter update laws (57)-(59) guarantees closed-loop system boundedness and asymptotic output tracking $\lim_{t\to\infty} e(t) = 0$ with $e(t) = y(t) - y_m(t)$.

Proof: (outline) The proof of this stability theorem can be established through using a unified framework. Because the control input $v(t)$ described in (43) depends on the state $x(t)$, it first needs to be expressed by using the system output $y(t)$ through establishing the state observer of the plant:

$$\dot{\hat{x}}(t) = (A - LC)\hat{x}(t) + Bv(t) + B_d d(t) + Ly(t) \tag{63}$$

where $L \in R^{n\times M}$ is a gain matrix such that $A - LC$ is stable, which is possible (AC) is assumed to be detectable. Hence we have

$$\begin{aligned} v(t) =& \Phi_1^T(t)\omega_1(t) + \Phi_2^T(t)\omega_2(t) \\ & + \Phi_{3d}^T(t)\omega_3(t) + K_2(t)r(t) + \Phi_3(t)f(t), \end{aligned} \tag{64}$$

where $\Phi_1^T(t)$, $\Phi_2^T(t)$, $\Phi_{3d}^T(t)$, $K_2(t)$ and $\Phi_3(t)$ are adaptive estimates of the corresponding nominal controller parameters and

$\omega_1(t) = \frac{a(s)}{\Lambda(s)}[v](t)$, $\omega_2(t) = \frac{a(s)}{\Lambda(s)}[y](t)$,
$\omega_3(t) = \frac{b(s)}{\Lambda(s)}[f](t)$,
with $a(s) = [I_M \quad sI_M \quad \dots \quad s^{n-1}I_M]^T$, $b(s) = [I_q \quad sI_q \quad \dots \quad s^{n-1}I_q]^T$, and $\Lambda(s)$ being a chosen monic stable polynomial of degree n, which has the same eigenvalues with $A - LC$. Then, introducing the fictitious filters for the plant $y(t) = C(sI - A)^{-1}Bv(t) + C(sI - A)^{-1}B_d d(t)$ and using series transformations, the control input described as (64) is transformed into the form

$$\begin{aligned} v(t) =& G_{11}(s,\cdot)[\bar{y}](t) + G_{12}(s,\cdot)[r](t) \\ & + G_{13}(s,\cdot)[f](t) + G_{14}(s,\cdot)[f_p](t), \end{aligned} \tag{65}$$

where $\bar{y}(t) = h(s)[y](t)$ ($h(s)$ is given below (48)) and $G_{11}(s,\cdot), G_{12}(s,\cdot), G_{13}(s,\cdot)$, and $G_{14}(s,\cdot)$ are proper stable operators with finite gains. Furthermore, a filtered version of the output signal $y(t)$ is expressed in a feedback framework:

$$\begin{aligned} \|\bar{y}(t)\| \leq & x_0 + \beta_1 \int_0^t e^{-\alpha_1(t-\tau)} x_1(\tau) \\ & \times \left(\int_0^\tau e^{-\alpha_2(\tau-\omega)} \|\bar{y}(\bar{\omega})\| d\bar{\omega}\right) d\tau, \end{aligned} \tag{66}$$

for some $\beta_1, \alpha_1, \alpha_2 > 0$, and $x_1(t) = \|\dot{\Phi}(t)\| + \|\varepsilon(t)\| m(t) \in L^2 \cap L^\infty$. Applying the small gain lemma to Eq.(60), we conclude that $\bar{y}(t) \in L^\infty$, and so $y(t), v(t) \in L^\infty$. Thus, the signals satisfy $\bar{\omega}(t), \zeta(t), \xi(t), m(t), \varepsilon \in L^\infty$. Furthermore, $\theta_i, \frac{\varepsilon(t)}{m(t)}, \dot{\Phi}(t), \dot{\Psi}(t) \in L^2$ (Lemma 4) are satisfied, and in turn $\xi(t)$ and $e(t) = y(t) - y_m(t)$, such that $e(t) = y(t) - y_m(t)$ converges to zero.

4. SIMULATION STUDY

In this section, the proposed LQR based MRAC scheme is applied to an aircraft control system model with disturbance. Through simulation evaluation, we give comparison among the LQR method, the PMRAC augmented method, single MRAC method and the proposed LQR based MRAC method under the disturbance situation. Different simulation results are given to illustrate the effectiveness of our proposed turbulence compensation method.

4.1. Aircraft system model

Our simulation model is chosen to represent lateral-directional motion of a conventional aircraft derived in [35], with the rudder δ_r primary control the yaw rate r and the sideslip angle β. The ailerons changing its roll rate p and the bank angle φ .

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\beta} \\ \dot{p} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ \frac{g}{V} & \frac{Y_\beta}{V} & \frac{Y_p}{V} & \frac{Y_r}{V} - 1 \\ 0 & L_\beta & L_p & Y_r \\ 0 & N_\beta & N_p & N_r \end{pmatrix} \begin{pmatrix} \varphi \\ \beta \\ p \\ r \end{pmatrix}$$

$$+\begin{pmatrix} 0 & 0 \\ \frac{Y_{\delta_a}}{V} & \frac{Y_{\delta_r}}{V} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \end{pmatrix}\begin{pmatrix} \delta_a \\ \delta_r \end{pmatrix}. \tag{67}$$

The following example for a small passenger aircraft in a cruise configuration, typical values of these parameters are

$$A_p = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.0487 & -0.0829 & 0 & -1 \\ 0 & -4.546 & -1.699 & 0.1717 \\ 0 & 3.382 & -0.0654 & -0.0893 \end{bmatrix},$$

$$B_p = \begin{bmatrix} 0 & 0 \\ 0 & 0.0116 \\ 27.276 & 0.5758 \\ 0.3952 & -1.362 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

4.2. Controller design

The baseline controller is

$$u_{bl} = K_x^T x. \tag{68}$$

After several design iterations, we have selected *LQR* diagonal

$$Q = diag\begin{pmatrix} 1 & 10 & 0.1 & 5 \end{pmatrix}, R = I_{2\times2}. \tag{69}$$

The resulting baseline *LQR* state feedback solution is

$$K_x^T = \begin{bmatrix} -1.0064 & -0.045 & -0.361 & 0.027 \\ -0.0823 & -1.3869 & -0.0459 & 2.552 \end{bmatrix}.$$

For the aircraft system, the transfer function, $G_p(s) = C(sI-A)^{-1}B$ has stable zeros: s1=-4.507, s2=-0.91, s3=-0.5685, and is strictly proper and full rank. The interactor matrix is chosen as

$$\xi(s) = diag\left\{ s+1 \quad (s+1)^2 \right\}. \tag{70}$$

The high-frequency matrix

$$K_p = \lim_{s\to\infty} \xi_m(s) G_p(s) = \begin{bmatrix} 2.2673 & 8.8455 \\ -0.0902 & -0.3521 \end{bmatrix}, \tag{71}$$

is finite and nonsingular and the matrix

$$K_d = \lim_{s\to\infty} \xi_m(s) G_d(s) = \begin{bmatrix} -0.0386 & -0.0526 \\ -0.2481 & -0.6246 \end{bmatrix}, \tag{72}$$

is finite. We choose $h(s) = \frac{1}{(s+1)^2}$.From $G_0(s)$ and $G_d(s)$, we can obtain

$$\lim_{s\to\infty} Z_0^{-1}(s) Z_d(s) = \begin{bmatrix} -0.017 & -0.0064 \\ 2.7493 & 1.7739 \end{bmatrix}, \tag{73}$$

this means relative degree condition Assumption 4 is ensured. The related gain parameters in adaptive laws (57)-(59) are chosen as

$$\begin{aligned} &P_\theta = 5, P_\phi = diag\left\{ 0.5 \quad 0.5 \right\}, \\ &P = diag\left\{ 1 \quad 1 \right\}. \end{aligned} \tag{74}$$

4.3. Simulation results

The LQR method, PMRAC augmented method, single MRAC method, LQR based MRAC method are simulated and comparisons are systematically presented. Three sets of simulation results are provided below. Case 1 is designed for comparison on the four controllers with reduction on control effectiveness and unmatched disturbance turned on. Case 2 and 3 are designed to show the effectiveness of the proposed LQR based MRAC augment controller.

For simulation studies, the initial state is chosen as $x_0(t) = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix}$ and the initial parameter values are set as 70% of their true values. We have selected the following control effectiveness $\Lambda = 0.5 I_{2\times2}$, the constant unknown coefficient is $\Gamma = \begin{pmatrix} 4A_p(2,2) & 2A_p(2,3) & 2A_p(2,4) \\ 4A_p(2,1) & 2A_p(3,3) & 2A_p(3,4) \end{pmatrix}^T$ and the known regressor vector is $\Omega(x) = \begin{pmatrix} \beta & p & r \end{pmatrix}^T$.

The unmatched disturbance is defined in [36] as

$$\begin{aligned} b_{\varphi_w} &= \begin{bmatrix} 0 & -0.0487 & 0 & 0 \end{bmatrix}^T, \\ b_{\beta_w} &= \begin{bmatrix} 0 & 0.0829 & 4.546 & 3.382 \end{bmatrix}^T. \end{aligned} \tag{75}$$

Two types of disturbance are described the constant turbulence and the time-varying turbulence.

1) We consider the constant roll and slide angle wind velocity disturbance as below:

$\varphi_w = 1crad/s, \beta_w = 1crad/s, \quad ('crad'is'centrad')$.

2) We time-varying roll and slide angle wind velocity disturbance as below:

$\varphi_w = 2\sin(0.2t)\ crad/s$,

$\beta_w = 3\sin(0.2t) + 2\sin(0.5t)\ crad/s$,

where ω_1, ω_2 and ω_3 are randomly chosen as 0.2, 0.3 and 0.5 for the numerical study, respectively. All the magnitudes of the preceding disturbances are unknown to the adaptive controller.

Case 1 is for time-varying tracking with reference step input when the uncertainty parameters and constant roll and slide angle wind velocity disturbance occur. As shown in Fig. 1 and Fig. 2, the baseline controller is able to stabilize the perturbed dynamics and the tracking performance is unacceptable. The aileron and rudder deflections exhibit the unwanted oscillations. As shown in Fig. 3 and Fig. 4, the LQR augment with PMRAC method was able to stabilize the perturbed dynamics. The corresponding aileron and rudder deflections are reasonable and well within the actuator capabilities. But there is a tracking error in response to unmatched uncertainty. As shown in Fig. 5 and Fig. 6, the single MRAC method was able to

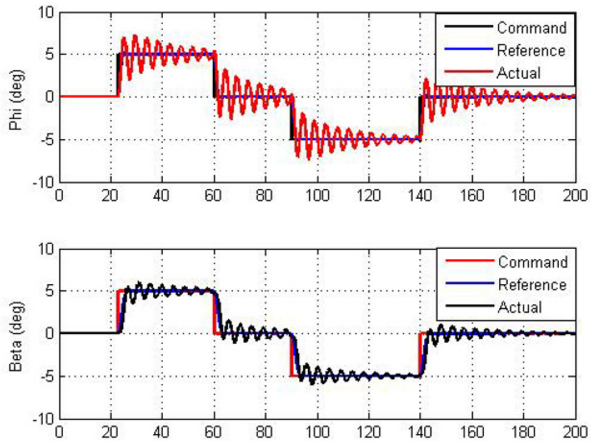

Fig. 1. LQR controller response with disturbance.

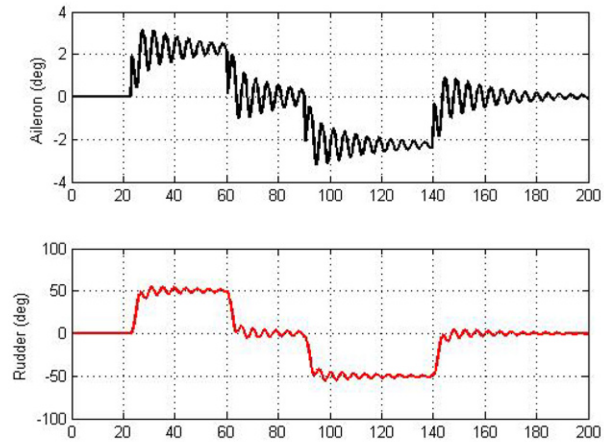

Fig. 2. LQR control signal.

stabilize the perturbed dynamics and recovers the desired closed-loop tracking performance. But the tracking transient of this controller is not well performed. As shown in Fig. 7 and Fig. 8, the proposed new LQR based MRAC method was able to stabilize the perturbed dynamics and recovers the desired closed-loop tracking performance and the transient of this controller is well performed.

Case 2 is for the corresponding time-varying tracking with time-varying input $r(t) = \begin{bmatrix} \sin(0.3t) & 0.5\sin(2t) \end{bmatrix}^T$, when the uncertainty parameters and constant roll and slide angle wind velocity disturbance occur. As shown in Fig.9 and Fig.10, the proposed new LQR based MRAC method was able to stabilize the perturbed dynamics and recovers the desired closed-loop tracking performance.

Case 3 is for corresponding time-varying tracking tracking with time-varying input $r(t) = \begin{bmatrix} \sin(0.3t) & 0.5\sin(2t) \end{bmatrix}^T$, when time-varying roll and slide angle wind velocity disturbance occur. As shown in Fig. 11 and Fig. 12, we can also draw the same conclusions as Case 2.

From the simulations above, the proposed new LQR based MRAC method shows great effectiveness in the present of matched and unmatched disturbance.

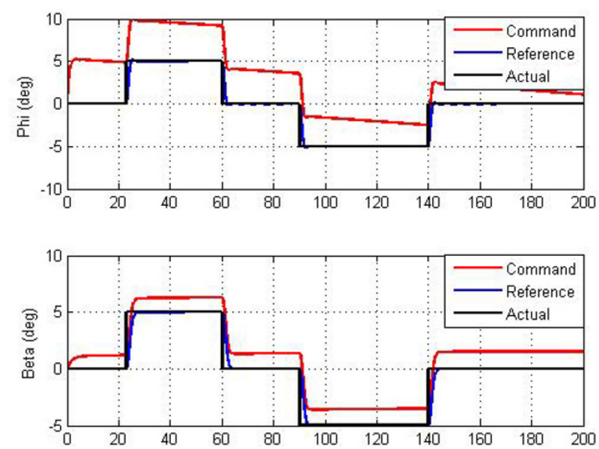

Fig. 3. PMRAC response with disturbance.

5. CONCLUSIONS

In this paper, disturbance rejection algorithm based on MRAC augmentation of linear quadratic regulator controller (LQR) is investigated for uncertain turbulence disturbances. The direct adaptive state feedback optimal controller based on an LDU gain decomposition parameterization is designed to solve the turbulence compensation problem to enhance control performance. Under the proposed control techniques, the bounded stability is achieved and the controller is able to remain within tight bounds on disturbance. Finally, simulation results are presented to illustrate the effectiveness of the MRAC augmentation of LQR controller. This paper has established disturbance rejection method in theory and demonstrated via simulation results from application to a linearized aircraft models around the equilibrium points. However, this design is still needed to apply to the nonlinear aircraft dynamics and non-minimum phase system. Our future research will address modifications of this algorithm to expand such operation domains.

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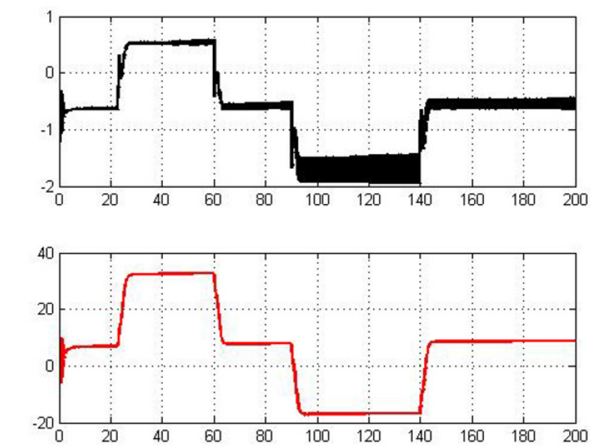

Fig. 4. PMRAC control signal.

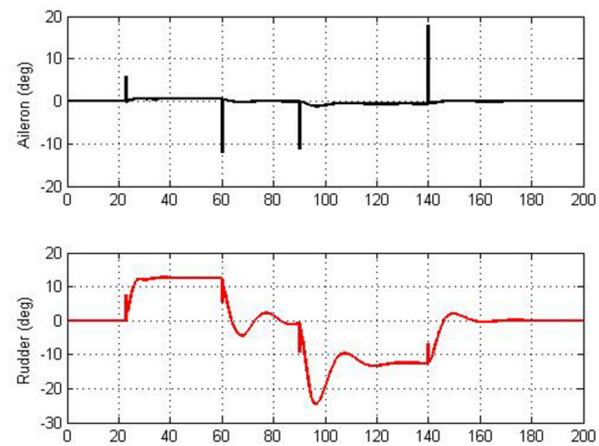

Fig. 6. Single MRAC control signal.

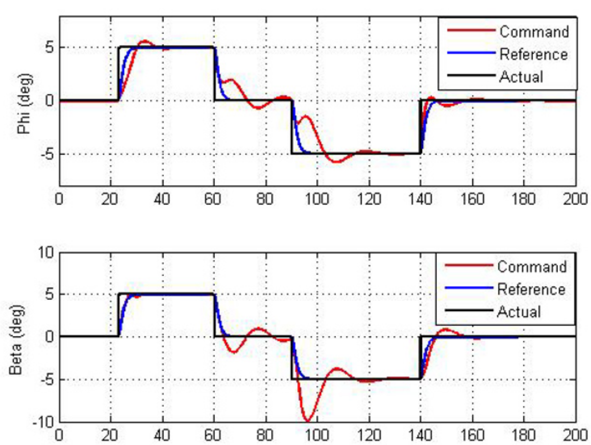

Fig. 5. Single MRAC response with disturbance.

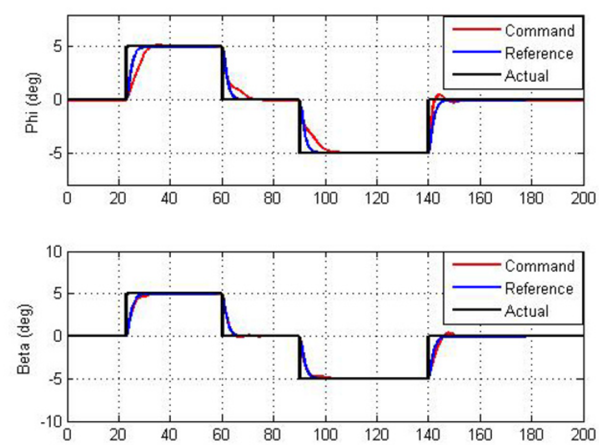

Fig. 7. New LQR based MRAC response with disturbance.

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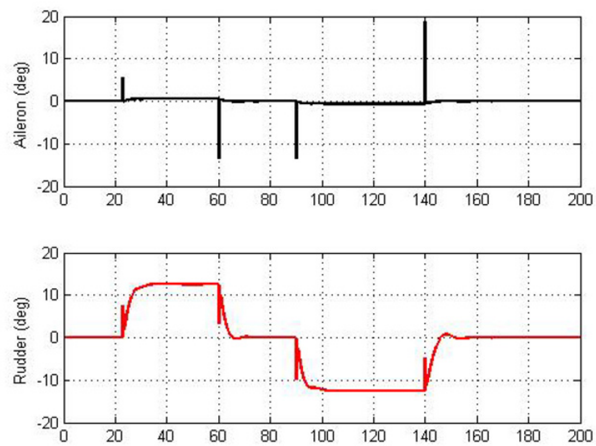

Fig. 8. New LQR based MRAC control signal.

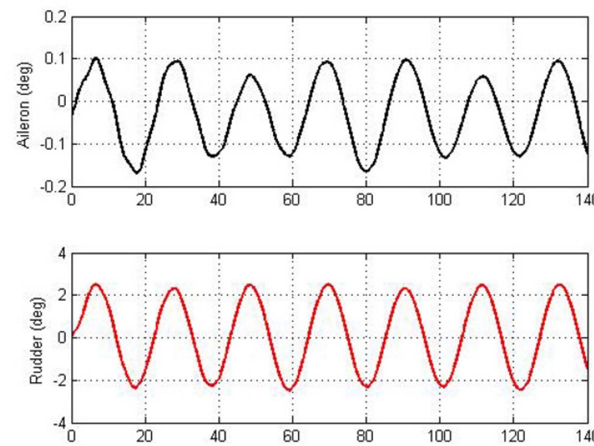

Fig. 10. New LQR based MRAC with time-varying input in the constant disturbances.

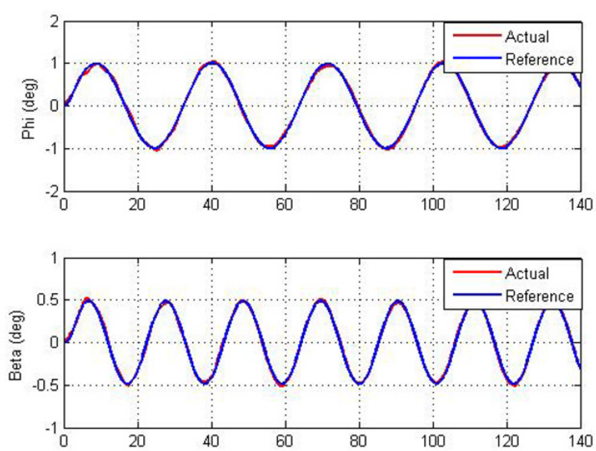

Fig. 9. New LQR based MRAC with time-varying input response in the constant disturbances.

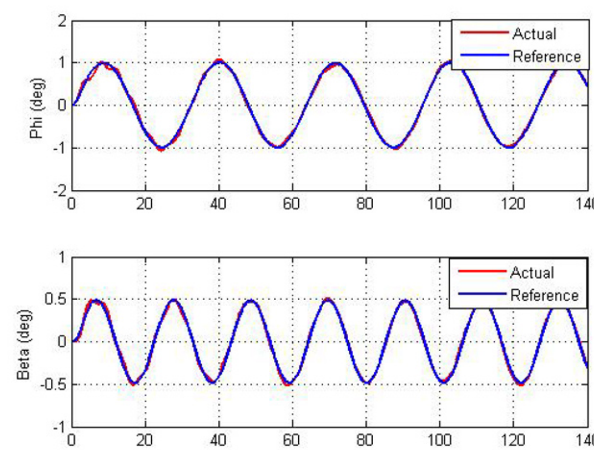

Fig. 11. New LQR based MRAC with time-varying response in the time-varying disturbances.

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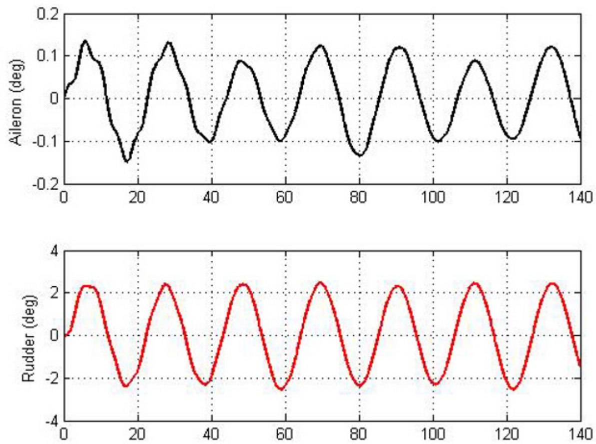

Fig. 12. New LQR based MRAC with time-varying response in the time-varying disturbances.

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